Algebra2/Trig
Ch4 Polynomial Functions – Review for Final

Name ____________________________
Period ______

1. Recall the definition of a polynomial: $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ where the exponents are whole numbers and the coefficients are real.
   a) Write 3 examples of functions that are polynomials.
   b) Write 3 examples of functions that are NOT polynomials.

2. If $f(x)$ is a polynomial of degree 3 and $g(x)$ is a polynomial of degree 2, state the degree of each of the following:
   a) $f(x)+g(x)$ 3
   b) $(f \cdot g)(x)$ 5
   c) $\dfrac{f(x)}{g(x)}$ 1

3. Recall that if a function has an irrational zero, it will also have the conjugate of the irrational zero as a zero. Write a polynomial function of least degree that has rational coefficients, a leading coefficient of 1 and the zeros 2 and $1+\sqrt{2}$. $f(x) = x^3 - 4x^2 + 3x + 2$

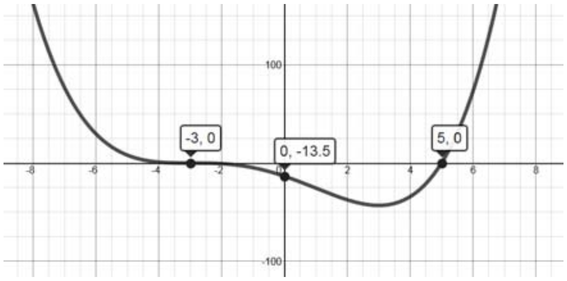
4. Describe the transformations on $f(x)$ represented by $g(x)$.
   a) $f(x) = x^3$ $g(x) = 2(x-3)^3$ Vertical stretch by 2; Shift right by 3
   b) $f(x) = x^4$ $g(x) = (2x-3)^4$ Shift right by 3; Then horizontal shrink by ½

5. Recall the definition for even functions is $f(x) = f(-x)$ (symmetric about the y-axis) and odd functions is $f(x) = -f(-x)$ (symmetric about the origin). Determine if each polynomial is even, odd or neither.
   a) $y = x^3$ odd
   b) $y = (x-1)^3$ neither
   c) $y = x^3 - 1$ neither
   d) $y = x^2$ even
   e) $y = (x-1)^2$ neither
   f) $y = x^2 - 1$ even

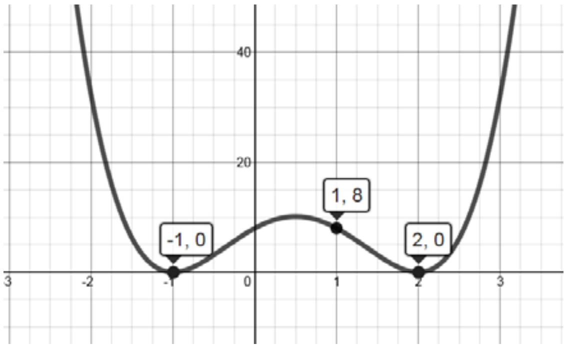
6. When looking for when a function takes on a certain output, we are looking for the x-values that give the y-coordinate equal to the output. Given the polynomial $f(x) = 2(x-1)^2(x+4)$, find the values of x such that $f(x) = 12$.
   a) From looking at the graph of $f(x)$, how many solutions are there? 3 solutions
   b) If possible, find the solutions algebraically. -3.73, -.27, 2
   c) Find all solutions graphically with the aid of a graphing calculator or Desmos. -3.73, -.27, 2

7. When given a graph of a polynomial, we can write the polynomial in factored form (aka intercept form) by noticing where the x-intercepts are. $f(x) = a(x-b)^n(x-c)^m \ldots$ If the graph passes through the x-axis at x = b, we know the power of the factor (x-b) is odd. If the graph is tangent to the x-axis at x = c, we know the power of

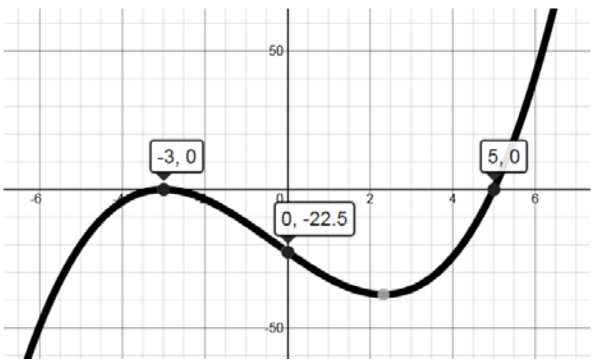
the factor (x-c) is even. The a value, which is the vertical stretch, can be solved for by plugging in any point other than the x-intercepts. Write the equation for each polynomial pictured. Give your answer in factored form.

a) 

$f(x)=.1(x-5)(x+3)^3$

b) 

$f(x)=2(x+1)^2(x-2)^2$

c) 

$f(x)=.5(x-5)(x+3)^2$

8. When given a polynomial function in expanded form and asked to factor completely, our goal is to write the polynomial as a product of linear factors. We continue factoring until each factor is prime. Review all techniques of factoring: factor by grouping, common factor, quadratic in form, the Rational Root Theorem.

a) Factor using the factor by grouping technique. Look for the common binomial.

i) $f(x)=x^3+3x^2-x-3$ $(x-1)(x+1)(x+3)$

ii) $f(x)=x^3+2x^2-4x-8$ $(x-2)(x+2)^2$

b) Factor by first noticing the common factor and pulling it out. Then see if you can factor any further.

i) $f(x)=3x^3-3x^2-18x$ $3x(x-3)(x+2)$

ii) $f(x)=x^4+2x^3-25x^2-50x$ $x(x-5)(x+5)(x+2)$

c) Factor by treating the polynomial as being quadratic in form.

i) $f(x)=x^4-5x^2+4$ $(x-2)(x+2)(x-1)(x+1)$

ii) $f(x)=x^6-2x^3-48$ $(x^3+6)(x-2)(x^2+2x+4)$

d) Factor by first listing all the possible factors according to the Rational Root Theorem. Then, use synthetic division to find which possible factors give a remainder of zero.

i) $f(x)=3x^4-39x^2+36x$ $3x(x-3)(x-1)(x+4)$

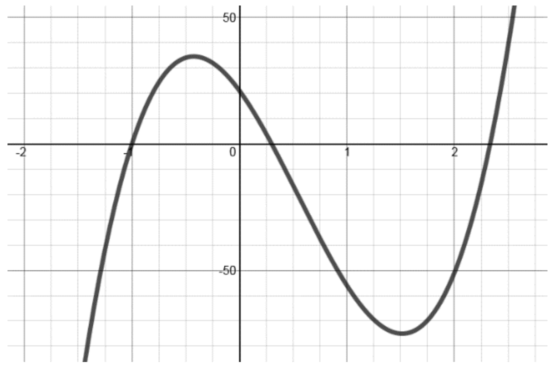
ii) $f(x)=2x^3+9x^2-11x-60$ $2(x-2.5)(x+3)(x+4)$

9. If we know a zero or two of a polynomial, that can help us to factor the polynomial equation. If we know the approximate location of a zero, we can narrow down our list of possible factors from the rational root theorem. Factor the following polynomials with the aid of the graphs and tables provided.

a) $f(x) = 6x^3 + 49x^2 + 88x - 35$ b) $f(x) = 30x^3 - 49x^2 - 58x + 21$

$f(x) = (3x-1)(2x+7)(x+5)$ $f(x) = (10x-3)(x+1)(3x-7)$

x	Y
-4	13
-3	-20
-2	-63



10. If you are given a binomial raised to a power and your goal is to expand it, Pascal's Triangle is a good way to start. Expand each of the following binomials and write in standard form.

a) $y = (2x+3z)^3$ $y = 8x^3 + 36x^2z + 54xz^2 + 27z^3$

b) $y = (p-2q)^4$ $y = p^4 - 8p^3q + 24p^2q^2 - 32pq^3 + 16q^4$

11. The overall degree of the polynomial, paired with the sign of the leading coefficient, can tell you the end behavior of the polynomial. Recall that if the degree of the polynomial is even, the function goes up to infinity on both ends of the graph. $f(x) =$ If the degree of the polynomial is odd, the function will go up to infinity on the right and down to negative infinity on the left. However, if there is a negative sign in the leading coefficient, the graph will flip over the x-axis. State the end behavior of each of the following functions.

a) $f(x) = 2x^4 - 3x + 1$ ($\nwarrow$, $\nearrow$)

b) $f(x) = 4x^3 - 1$ ($\swarrow$, $\nearrow$)

c) $f(x) = -2x^5 - 3x^4 + 4$ ($\nwarrow$, $\searrow$)

12. A turning point on a polynomial is considered a local minimum if the point is the lowest in the neighborhood around it, and a local maximum if the point is the highest in the neighborhood around it. When the slope is negative, the polynomial is said to be decreasing and when the slope is positive, the polynomial is said to be increasing. We can find the turning points by graphing the polynomial on a graphing calculator or Desmos and using the max/min feature of the calculator or Desmos. Graph each polynomial and give the intervals that the function is increasing and decreasing. List the local maximums and local minimums.

a) $f(x) = \frac{1}{3}(x+3)(x-1)(x+1)$

b) $f(x) = (x-2)(x+3)^2(x-1)$